

$$\begin{aligned}
& \sum_{k=-\infty}^{\infty} \cos(k\pi/n) \cos(k\pi/s) \\
&= (1/2) \sum_{k=-\infty}^{\infty} \{ \cos[k\pi(1/n+1/s)] + \cos[k\pi(1/n-1/s)] \} \\
&= 1 + \sum_{k=1}^{\infty} \{ \cos[k\pi(1/n+1/s)] + \cos[k\pi(1/n-1/s)] \} \\
&\quad \downarrow \\
&\quad \sum_{k=1}^N \cos(ka) = \frac{\sin[(N+1/2)a]}{2\sin(a/2)} - \frac{1}{2} \\
&= (\infty + 1/2) \delta_{ns}
\end{aligned}$$


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$$\begin{aligned}
& \sum_{k=-\infty}^{\infty} \sin(k\pi/n) \sin(k\pi/s) \\
&= (1/2) \sum_{k=-\infty}^{\infty} \{ \cos[k\pi(1/n-1/s)] - \cos[k\pi(1/n+1/s)] \} \\
&= \sum_{k=1}^{\infty} \{ \cos[k\pi(1/n-1/s)] - \cos[k\pi(1/n+1/s)] \} \\
&= (\infty + 1/2) \delta_{ns}
\end{aligned}$$