

$$a_n(\chi) = \sum_{k=-\infty}^{\infty} \chi_k \cos(k\pi/n)$$

$$b_n(\chi) = \sum_{k=-\infty}^{\infty} \chi_k \sin(k\pi/n)$$

$$\Phi(\chi) = F(a_1(\chi), b_1(\chi); a_2(\chi), b_2(\chi); a_3(\chi), b_3(\chi); \dots)$$


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$$\begin{aligned} \frac{\partial \Phi(\chi)}{\partial \chi_k} &= \sum_{n=1}^{\infty} \left[ \frac{\partial a_n(\chi)}{\partial \chi_k} \cdot \frac{\partial F}{\partial a_n} + \frac{\partial b_n(\chi)}{\partial \chi_k} \cdot \frac{\partial F}{\partial b_n} \right] \\ &= \sum_{n=1}^{\infty} \left[ \frac{\partial F}{\partial a_n} \cos(k\pi/n) + \frac{\partial F}{\partial b_n} \sin(k\pi/n) \right] \end{aligned}$$

$$\begin{aligned} &\sum_{k=-\infty}^{\infty} \frac{\chi_{k-1} - \chi_k}{1/\alpha} \cdot \frac{\partial \Phi(\chi)}{\partial \chi_k} \\ &= \alpha \sum_{n=1}^{\infty} \left[ \frac{\partial F}{\partial a_n} \sum_{k=-\infty}^{\infty} (\chi_{k-1} - \chi_k) \cos(k\pi/n) \right. \\ &\quad \left. + \frac{\partial F}{\partial b_n} \sum_{k=-\infty}^{\infty} (\chi_{k-1} - \chi_k) \sin(k\pi/n) \right] \end{aligned}$$