

宇田方程式：

$$\frac{i\hbar}{\alpha} \cdot \frac{\partial \Phi[\chi(\square - \varepsilon)]}{\partial \varepsilon} \Big|_{\varepsilon=0} = \frac{1}{2m} \int_{-\infty}^{\infty} dt \left[\frac{-i\hbar}{\alpha} \cdot \frac{\delta}{\delta \chi(t)} \right]^2 \Phi[\chi]$$

$$\chi(\square - \varepsilon) : t \rightarrow \chi(t - \varepsilon)$$

離散化：

$$\chi \rightarrow (\dots, \chi_{-2}, \chi_{-1}, \chi_0, \chi_1, \chi_2, \dots)$$

$$\chi_k \equiv \chi(k/\alpha)$$

$$\Phi[\chi] \rightarrow \Phi(\dots, \chi_{-2}, \chi_{-1}, \chi_0, \chi_1, \chi_2, \dots)$$

$$\begin{aligned} \frac{\partial \Phi[\chi(\square - \varepsilon)]}{\partial \varepsilon} \Big|_{\varepsilon=0} &\rightarrow \frac{\Phi[\chi(\square - 1/\alpha)] - \Phi[\chi]}{1/\alpha} \\ &= \frac{\Phi(\chi_{\square-1}) - \Phi(\chi)}{1/\alpha} \\ &= \sum_{k=-\infty}^{\infty} \frac{\chi_{k-1} - \chi_k}{1/\alpha} \cdot \frac{\partial \Phi(\chi)}{\partial \chi_k} \end{aligned}$$

$$\int_{-\infty}^{\infty} dt \left[\frac{-i\hbar}{\alpha} \cdot \frac{\delta}{\delta \chi(t)} \right]^2 \Phi(\chi) \rightarrow \frac{1}{\alpha} \sum_{k=-\infty}^{\infty} \left(-i\hbar \frac{\partial}{\partial \chi_k} \right)^2 \Phi(\chi)$$