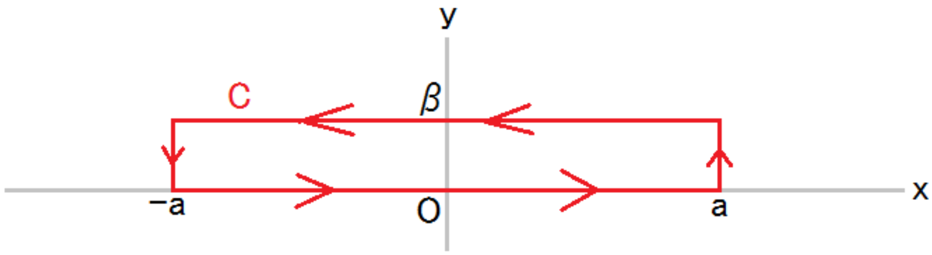
$$\delta(z)=\frac{1}{2\pi}\int_{-\infty}^{\infty} dp\,\exp(ipz) \qquad \int_{-\infty}^{\infty} dx\,f(x)\,\delta(x-\alpha-i\beta)=f(\alpha+i\beta)$$



$$0=\int_C dz\,f(z)\,\delta(z-\alpha-i\beta)$$

$$=\int_{-a}^{a} dx\,f(x)\,\delta(x-\alpha-i\beta)$$

$$+\int_0^{\beta} idy\,f(a+iy)\,\delta(a+iy-\alpha-i\beta)$$

$$+\int_a^{-a} dx\,f(x+i\beta)\,\delta(x+i\beta-\alpha-i\beta)$$

$$+\int_{\beta}^{0} idy\,f(-a+iy)\,\delta(-a+iy-\alpha-i\beta)$$

$$=\int_{-a}^{a} dx\,f(x)\,\delta(x-\alpha-i\beta)$$

$$-\int_{-a}^{a} dx\,f(x+i\beta)\,\delta(x-\alpha)$$

$$\left.\begin{aligned}&+\ i\int_0^{\beta} dy\,f(a+iy)\,\delta(a+iy-\alpha-i\beta)\\ &-\ i\int_0^{\beta} dy\,f(-a+iy)\,\delta(-a+iy-\alpha-i\beta)\end{aligned}\right]0$$

$\lvert x\rvert\to\infty$ で $f(x+iy)\to$?
$a=2n\pi\to\pm\infty$

$$\therefore\quad \int_{-a}^{a} dx\,f(x)\,\delta(x-\alpha-i\beta)=\int_{-a}^{a} dx\,f(x+i\beta)\,\delta(x-\alpha)$$

$$=f(\alpha+i\beta)$$