宇田方程式	$\int_{-\infty}^{\infty} dt \left\{ \frac{1}{2m}\left[-\frac{i\hbar}{\alpha}\frac{\delta}{\delta\chi(t)} - m\frac{d\chi(t)}{dt}\right]^2 - \frac{m}{2}\left[\frac{d\chi(t)}{dt}\right]^2\right\}\Phi[\chi]=0$
ハミルトニアン	$H = H_0 + H_1 \begin{cases} H_0 \equiv \frac{1}{2m\varepsilon\alpha^2}\sum_{k=-\infty}^{\infty}[(P^k)^2 + m\alpha(P^kX^k+X^kP^k)] \\ H_1 \equiv -\frac{1}{\varepsilon\alpha}\sum_{k=-\infty}^{\infty} P^kX^{k+1}\end{cases}$

自由粒子1個の定常状態

$$\underbrace{\left[\left(-i\hbar\frac{d}{dx}\right)^2 - i\hbar m\alpha\left(1+2x\frac{d}{dx}\right)\right]}_{(P)^2+m\alpha(PX+XP)}u(x)=e\,u(x)$$

$$u(x)=\sum_{n=0}^{\infty} a_n x^n \quad \begin{cases} 2\hbar^2 a_2 + (e+i\hbar m\alpha)a_0 = 0 \\ \hbar^2(n+2)(n+1)a_{n+2} + [e+i\hbar m\alpha(2n+1)]a_n = 0 \ (n \geqq 1)\end{cases}$$

$$a_{2n}(e)=\frac{1}{(2n)!}\left(\frac{-4im\alpha}{\hbar}\right)^n \frac{\Gamma(n+1/4+e/(4i\hbar m\alpha))}{\Gamma(5/4+e/(4i\hbar m\alpha))}\frac{e+i\hbar m\alpha}{4i\hbar m\alpha}a_0(e)$$

$$a_{2n+1}(e)=\frac{1}{(2n+1)!}\left(\frac{-4im\alpha}{\hbar}\right)^n \frac{\Gamma(n+3/4+e/(4i\hbar m\alpha))}{\Gamma(3/4+e/(4i\hbar m\alpha))}a_1(e)$$

$$\Gamma(n+1)=n\Gamma(n)$$