$$\frac{d^2x}{d\tau^2}=\frac{1}{\varepsilon\alpha}A\frac{dx}{d\tau}+\frac{1}{\varepsilon^2\alpha^2}Bx$$

$$A_{jk}=\delta_{j-1,k}-\delta_{j+1,k}$$

$$B_{jk}=2\delta_{jk}-\delta_{j-1,k}-\delta_{j+1,k}$$

$$\frac{d^2x}{d\tau^2}=\frac{1}{\varepsilon\alpha}A\frac{dx}{d\tau}+kBx$$

$$x=x_0+kx_1+k^2x_2+\cdot\cdot\cdot$$

$k \ll 1$

$\frac{1}{\varepsilon\alpha}$は必ずしも小さくない。

$$\frac{d^2x_0}{d\tau^2}=\frac{1}{\varepsilon\alpha}A\frac{dx_0}{d\tau}$$

$$\frac{d^2x_1}{d\tau^2}=\frac{1}{\varepsilon\alpha}A\frac{dx_1}{d\tau}+Bx_0$$

$$\frac{dx_0}{d\tau}=a\exp\left(\frac{\tau}{\varepsilon\alpha}A\right)v,$$

$$x_0=a\varepsilon\alpha\exp\left(\frac{\tau}{\varepsilon\alpha}A\right)u+w,\qquad v=Au.$$

$\exp\left(\frac{\tau}{\varepsilon\alpha}A\right)$

無限次元実回転

$x^j=aj+b$

↓

$x'^j=aj+b'$

$$\frac{dx_1}{d\tau}=\exp\left(\frac{\tau}{\varepsilon\alpha}A\right)\int_c^\tau d\tau'\exp\left(-\frac{\tau'}{\varepsilon\alpha}A\right)Bx_0(\tau')$$

$$=\exp\left(\frac{\tau}{\varepsilon\alpha}A\right)\int_c^\tau d\tau'\exp\left(-\frac{\tau'}{\varepsilon\alpha}A\right)B\left[a\varepsilon\alpha\exp\left(\frac{\tau'}{\varepsilon\alpha}A\right)u+w\right]$$

$$=\exp\left(\frac{\tau}{\varepsilon\alpha}A\right)\int_c^\tau d\tau'\left[a\varepsilon\alpha Bu+\exp\left(-\frac{\tau'}{\varepsilon\alpha}A\right)Bw\right]\quad\because [A,B]_-=0$$

$$=\varepsilon\alpha\left\{\exp\left(\frac{\tau}{\varepsilon\alpha}A\right)\left[a(\tau-c)Bu+\exp\left(\frac{-c}{\varepsilon\alpha}A\right)w'\right]-w'\right\}\quad(Aw'=Bw)$$

$$\frac{d}{d\tau}(x_0+kx_1)$$
$$=\exp\left(\frac{\tau}{\varepsilon\alpha}A\right)\left[aAu+ak\varepsilon\alpha(\tau-c)Bu+k\varepsilon\alpha\exp\left(\frac{-c}{\varepsilon\alpha}A\right)w'\right]-k\varepsilon\alpha w'$$