

$$D(t_1 - t_2) \rightarrow \frac{i}{\hbar} \left[ -\frac{m}{2} \delta^{(2)}(t_1 - t_2) - \frac{k}{2} \delta(t_1 - t_2) \right]$$


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$$\begin{aligned} \Phi[X] &= \exp \int dt_1 \int dt_2 X(t_1) D(t_1 - t_2) X(t_2) \\ &\rightarrow \exp \left\{ \frac{i}{\hbar} \int dt \left[ \frac{m}{2} [\dot{X}(t)]^2 - \frac{k}{2} [X(t)]^2 \right] \right\} \end{aligned}$$


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$$\begin{aligned} &\int_0^T dt \left[ \frac{\delta}{\delta X(t)} \right]^2 \Phi[X] \\ &= [4 \int dt_1 \int dt_2 X(t_1) \int dt D(t_1 - t) D(t - t_2) X(t_2) \\ &\quad + 2TD(0)] \Phi[X] \\ &\rightarrow \left\{ 2TD(0) - \frac{1}{\hbar^2} \int_0^T dt \left[ m^2 [\ddot{X}(t)]^2 \right. \right. \\ &\quad \left. \left. - 2mk[\dot{X}(t)]^2 + k^2[X(t)]^2 \right] \right\} \Phi[X] \end{aligned}$$