

$$\begin{aligned}
& \frac{\delta}{\delta \chi(t)} \exp \left\{ (i/\hbar) S[\chi] \right\} \\
&= \lim_{\varepsilon \rightarrow 0} \frac{1}{\varepsilon} \left[\exp \left\{ (i/\hbar) S[\chi + \varepsilon \delta(\square - t)] \right\} \right. \\
&\quad \left. - \exp \left\{ (i/\hbar) S[\chi] \right\} \right] \\
&= \frac{d}{d\varepsilon} \left[\exp \left\{ (i/\hbar) S[\chi + \varepsilon \delta(\square - t)] \right\} \right]_{\varepsilon=0} \\
&= \frac{i}{\hbar} \frac{d S[\chi + \varepsilon \delta(\square - t)]}{d\varepsilon} \Bigg|_{\varepsilon=0} \exp \left\{ (i/\hbar) S[\chi] \right\} \\
&= \frac{i}{\hbar} \frac{\delta S[\chi]}{\delta \chi(t)} \exp \left\{ (i/\hbar) S[\chi] \right\}
\end{aligned}$$