

$$\frac{d}{dt} \int D\mathcal{X} \overline{\Phi[\mathcal{X}]} \mathcal{X}(t) \Phi[\mathcal{X}]$$

$$= \frac{d}{d\varepsilon} \int D\mathcal{X} \overline{\Phi[\mathcal{X}]} \mathcal{X}(t+\varepsilon) \Phi[\mathcal{X}] \Big|_{\varepsilon=0}$$

↓ 置換積分: $\mathcal{X}(t) = \eta(t-\varepsilon)$, $\int D\mathcal{X} = \int D\eta$

$$= \frac{d}{d\varepsilon} \int D\eta \overline{\Phi[\eta']} \eta(t) \Phi[\eta'] \Big|_{\varepsilon=0} ; \eta'(t) \equiv \eta(t-\varepsilon)$$

$$= \int D\eta \left[\frac{d}{d\varepsilon} \overline{\Phi[\eta']} \Big|_{\varepsilon=0} \right] \eta(t) \Phi[\eta]$$

$$+ \int D\eta \overline{\Phi[\eta]} \eta(t) \left[\frac{d}{d\varepsilon} \Phi[\eta'] \Big|_{\varepsilon=0} \right]$$

$$= \frac{\alpha}{-i\hbar} \int D\eta \overline{\Phi[\eta]} \eta(t) \int dt' \left(\frac{1}{2m} \left[\frac{i\hbar}{\alpha} \frac{\delta}{\delta\eta(t')} \right]^2 + V(\eta(t')) \right) \overline{\Phi[\eta]}$$

$$+ \frac{\alpha}{i\hbar} \int D\eta \overline{\Phi[\eta]} \eta(t) \int dt' \left(\frac{1}{2m} \left[\frac{-i\hbar}{\alpha} \frac{\delta}{\delta\eta(t')} \right]^2 + V(\eta(t')) \right) \Phi[\eta]$$

$$= \frac{-i\hbar}{2m\alpha} \int D\eta \overline{\Phi[\eta]} \left(\int dt' \left[\frac{\delta}{\delta\eta(t')} \right]^2 \eta(t) - \eta(t) \int dt' \left[\frac{\delta}{\delta\eta(t')} \right]^2 \right) \Phi[\eta]$$

